# The Hash Function JH 

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## 1 Introduction

This document specifies four hash algorithms - JH-224, JH-256, JH-384, and JH-512. The hash algorithms are very simple. They are efficient on many platforms ranging from one-bit processor (hardware) to 128-bit processor (SSE2 registers) since they are built on extremely simple components.

The JH hash functions are very efficient in software. With bit-slice implementation using SSE2, the speed of JH is about 16.8 clock cycles/byte on the Intel Core 2 Duo microprocessor running 64-bit operating system (using C implementation and Intel C++ compiler).

The memory required for the hardware implementation of JH hash functions is 1536 bits. With 256 additional memory bits, the round constants of JH can be generated on the fly. JH-224, JH-256, JH-384 and JH-512 share the same compression function, so it is very efficient to implement these four hash algorithms altogether in hardware.

JH is strong in security. Each message block is 64 bytes. A message block passes through the 35.5 -round compression function that involves 92164 -bit-to-4-bit Sboxes. We found that any differential trail in the compression function involves around 600 active Sboxes. The large number of active Sboxes ensures that JH is strong against differential attack [1].

This document is organized as follows. The specifications of JH are given in Sec. 3, 4, 5 and 6. The bit-slice implementation of JH is given in Sec. 7. Section 9 gives the security analysis of JH. The performance of JH is described in Sec. 10. The design rationale and advantage are given in Sec. 11 and Sec. 12, respectively. Sec. 13 concludes this document.

## 2 Efficient Differential Propagation Design

The compression function in JH is based on a bijective function. The Efficient Differential Propagation (EDP) design is used to design the bijective function in JH. EDP design uses the substitution-permutation network (SPN). The input bits are divided into $\alpha \times 2^{d}$ elements, and these elements form a $d$-dimensional array. In the linear layer of the $r$-th round, Maximum Distance Separable (MDS) code is applied along the ( $r \bmod d$ )-th dimension. We believe that such design is the simplest approach to achieve efficient differential propagation.

EDP design is not new. AES (Rijndael [6]) is based on EDP design with a two-dimensional array. However, Rijndael with 192-bit and 256-bit block sizes are not based on EDP design since MDS code is not applied to the dimension with 6 (192-bit block size) or 8 (256-bit block size) elements.

We use an eight-dimensional EDP design in the design of the bijective function in JH. The 1024 input bits to the bijective function are divided into $2^{8} 4$-bit elements, and these elements form an eight-dimensional array.

## 3 Definitions

### 3.1 Notations

The following notations are used in the JH specifications.
Word A group of bits.
$A^{i} \quad$ The $i^{\text {th }}$ bit in the word $A$. An $m$-bit word $A$ is represented as $A=A^{0}\left\|A^{1}\right\| A^{2}\|\cdots\| A^{m-1}$.

### 3.2 Parameters

The following parameters are used in the JH specifications.

| $C_{r}^{(d)}$ | The round constant words used in function $E_{d}$ with $0 \leq$ $r \leq 5 \times(d-1)$. Each $C_{r}^{(d)}$ is a $2^{d}$-bit constant word. |
| :---: | :---: |
| $d$ | The dimension of a block of bits. A $d$-dimensional block consists of $2^{d} 4$-bit elements. |
| $h$ | Number of bits in a hash value. $h=1024$. |
| $H^{(i)}$ | The $i^{\text {th }}$ hash value, with a size of $h$ bits. $H^{(0)}$ is the initial hash value; $H^{(N)}$ is the final hash value and is truncated to generate the message digest. |
| $H^{(i), j}$ | The $j^{\text {th }}$ bit of the $i^{\text {th }}$ hash value, where $H^{(i)}=$ $H^{(i), 0}\left\\|H^{(i), 1}\right\\| \cdots \\| M^{(i), h-1}$. |
| $\ell$ | Length of the message, $M$, in bits. |
| $m$ | Number of bits in a message block $M^{(i)} . m=512$. |
| M | Message to be hashed. |
| $M^{(i)}$ | Message block $i$, with a size of $m$ bits. |
| $M^{(i), j}$ | The $j^{\text {th }}$ bit of the $i^{\text {th }}$ message block, i.e., $M^{(i)}=$ $M^{(i), 0}\left\\|M^{(i), 1}\right\\| \cdots \\| M^{(i), m-1}$. |
| $N$ | Number of blocks in the padded message. |

### 3.3 Operations

The following operations are used in the JH specifications.

| $\&$ | Bitwise AND operation. |
| :--- | :--- |
| $\mid$ | Bitwise OR ("inclusive-OR") operation. |
| $\oplus$ | Bitwise XOR ("exclusive-OR") operation. |
| $\neg$ | Bitwise complement operation. |
| $\\|$ | Concatenation operation. |

## 4 Functions

The following functions are used in the JH specifications.

### 4.1 S-boxes

$S_{0}$ and $S_{1}$ are the 4 -bit-to-4-bit S-boxes being used in JH. Every round constant bit selects which Sboxes are used (similar to Lucifer [9]).

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $S_{0}(x)$ | 9 | 0 | 4 | 11 | 13 | 12 | 3 | 15 | 1 | 10 | 2 | 6 | 7 | 5 | 8 | 14 |
| $S_{1}(x)$ | 3 | 12 | 6 | 13 | 5 | 7 | 1 | 9 | 15 | 2 | 0 | 4 | 11 | 10 | 14 | 8 |

### 4.2 Linear transformation $L$

The linear transformation $L$ implements a $(4,2,3)$ Maximum Distance Separable (MDS) code over $G F\left(2^{4}\right)$. Here the multiplication in $G F\left(2^{4}\right)$ is defined as the multiplication of binary polynomials modulo the irreducible polynomial $x^{4}+x+1$. Denote this multiplication as ' $\bullet$ '.

Let $A, B, C$ and $D$ denote 4 -bit words. $L$ transforms $(A, B)$ into $(C, D)$ as

$$
(C, D)=L(A, B)=(5 \bullet A+2 \bullet B, 2 \bullet A+B) .
$$

More specifically, the bit-wise computation of $L$ is given as follows. Let $A, B, C$ and $D$ denote 4 -bit words, i.e., $A=A^{0}\left\|A^{1}\right\| A^{2} \| A^{3}, B=$ $B^{0}\left\|B^{1}\right\| B^{2}\left\|B^{3}, C=C^{0}\right\| C^{1}\left\|C^{2}\right\| C^{3}$, and $D=D^{0}\left\|D^{1}\right\| D^{2} \| D^{3}$. In polynomial form, $A$ is represented as $A^{0} x^{3}+A^{1} x^{2}+A^{2} x+A^{3} ; 2 \bullet A$ is given as $A^{1} x^{3}+A^{2} x^{2}+\left(A^{0}+A^{3}\right) x+A^{0}$. The function $(C, D)=L(A, B)$ is computed as:

$$
\begin{array}{ll}
D^{0}=B^{0} \oplus A^{1} ; \quad D^{1}=B^{1} \oplus A^{2} ; \\
D^{2}=B^{2} \oplus A^{3} \oplus A^{0} ; D^{3}=B^{3} \oplus A^{0} ; \\
C^{0}=A^{0} \oplus D^{1} ; \quad C^{1}=A^{1} \oplus D^{2} ; \\
C^{2}=A^{2} \oplus D^{3} \oplus D^{0} ; C^{3}=A^{3} \oplus D^{0} .
\end{array}
$$

### 4.3 Permutation $P_{d}$

$P_{d}$ is a simple permutation on $2^{d}$ elements. It is constructed from $\pi_{d}, P_{d}^{\prime}$ and $\phi_{d}$. Denote $2^{d}$ input elements as $A=\left(a_{0}, a_{1}, \cdots, a_{2^{d}-1}\right)$, and $2^{d}$ output elements as $B=\left(b_{0}, b_{1}, \cdots, b_{2^{d}-1}\right)$.

### 4.3.1 Permutation $\pi_{d}$

$\pi_{d}$ operates on $2^{d}$ elements. The computation of $B=\pi_{d}(A)$ is as follows:

$$
\begin{aligned}
& b_{4 i+0}=a_{4 i+0} \quad \text { for } i=0 \text { to } 2^{d-2}-1 \text {; } \\
& b_{4 i+1}=a_{4 i+1} \quad \text { for } i=0 \text { to } 2^{d-2}-1 \text {; } \\
& b_{4 i+2}=a_{4 i+3} \quad \text { for } i=0 \text { to } 2^{d-2}-1 \text {; } \\
& b_{4 i+3}=a_{4 i+2} \quad \text { for } i=0 \text { to } 2^{d-2}-1 \text {; }
\end{aligned}
$$

The permutation $\pi_{4}$ is illustrated in Fig. 1.


Figure 1: The permutation $\pi_{4}$

### 4.3.2 Permutation $P_{d}^{\prime}$

$P_{d}^{\prime}$ is a permutation on $2^{d}$ elements. The computation of $B=P_{d}^{\prime}(A)$ is given as follows:

$$
\begin{aligned}
b_{i} & =a_{2 i} \text { for } i=0 \text { to } 2^{d-1}-1 ; \\
b_{i+2^{N-1}} & =a_{2 i+1} \text { for } i=0 \text { to } 2^{d-1}-1 ;
\end{aligned}
$$

The permutation $P_{4}^{\prime}$ is illustrated in Fig. 2.


Figure 2: The permutation $P_{4}^{\prime}$

### 4.3.3 Permutation $\phi_{d}$

$\phi_{d}$ is a permutation on $2^{d}$ elements. The computation of $B=\phi_{d}(A)$ is given as follows:

$$
\begin{aligned}
b_{i} & =a_{i} & \text { for } i=0 \text { to } 2^{d-1}-1 ; \\
b_{2 i+0} & =a_{2 i+1} & \text { for } i=2^{d-1} \text { to } 2^{d}-1 ; \\
b_{2 i+1} & =a_{2 i+0} \quad & \text { for } i=2^{d-1} \text { to } 2^{d}-1 ;
\end{aligned}
$$

The permutation $\phi_{4}$ is illustrated in Fig. 3.

### 4.3.4 Permutation $P_{d}$

$P_{d}$ is the composition of $\pi_{d}, P_{d}^{\prime}$ and $\phi_{d}$ :

$$
P_{d}=\phi_{d} \circ P_{d}^{\prime} \circ \pi_{d}
$$

The permutation $P_{4}$ is illustrated in Fig. 4.


Figure 3: The permutation $\phi_{4}$


Figure 4: The permutation $P_{4}$

### 4.4 Round function $R_{d}$

The round function $R_{d}$ implements the Efficient Differential Propagation (EDP) design illustrated in Sec. 2. It consists of three layers: the Sbox layer, the linear transform layer and the permutation layer $P_{d}$. The input and output sizes of $R_{d}$ are $2^{d+2}$ bits. The $2^{d+2}$-bit input word is denoted as $A=\left(a_{0}\left\|a_{1}\right\| \cdots \| a_{2^{d}-1}\right)$, where each $a_{i}$ represents a 4 -bit word. The $2^{d+2}$-bit output word is denoted as $B=\left(b_{0}\left\|b_{1}\right\| \cdots \| b_{2^{d}-1}\right)$, where each $b_{i}$ represents a 4 -bit word. The $2^{d}$-bit round constant of the $r$-th round is denoted as $C_{r}^{(d)}=C_{r}^{(d), 0}\left\|C_{r}^{(d), 1} \cdots\right\| C_{r}^{(d), 2^{d}-1}$. Let each $v_{i}$ and $w_{i}(0 \leq i \leq$ $2^{d}-1$ ) represent a 4 -bit word. The computation of $B=R_{d}\left(A, C_{r}^{(d)}\right)$ is given as follows:

1. for $i=0$ to $2^{d}-1$,
\{

$$
\begin{aligned}
& \text { if } C_{r}^{(d), r}=0 \text {, then } v_{i}=S_{0}\left(a_{i}\right) ; \\
& \text { if } C_{r}^{(d), i}=1 \text {, then } v_{i}=S_{1}\left(a_{i}\right) ;
\end{aligned}
$$

\}
2. $\left(w_{2 i}, w_{2 i+1}\right)=L\left(v_{2 i}, v_{2 i+1}\right) \quad$ for $0 \leq i \leq 2^{d-1}-1$;
3. $\left(b_{0}, b_{1}, \cdots, b_{2^{d}-1}\right)=P_{d}\left(w_{0}, w_{1}, \cdots, w_{2^{d}-1}\right)$;

Two rounds of $R_{4}$ are illustrated in Fig. 5.


Figure 5: Two rounds of $R_{4}$ (constant bits not shown)

### 4.5 Bijective function $E_{d}$

$E_{d}$ is based on the $d$-dimensional EDP design. It is constructed from $5(d-$ 1) rounds of $R_{d}$, plus an additional Sbox layer. The $2^{d+2}$-bit input and output are denoted as $A$ and $B$, respectively. Let each $Q_{r}$ denote a $2^{d+2}$-bit word for $0 \leq r \leq 4 d+1$, and $Q_{r}=\left(q_{r, 0}\left\|q_{r, 1}\right\| \cdots \| q_{r, 2^{d}-1}\right)$, where each $q_{r, i}$ denotes a 4 -bit word. Let $R_{d}^{*}$ denote the round function $R_{d}$ with the linear transformation and permutation being removed. Let $d^{\prime}=d-1$. The computation of $B=E_{d}(A)$ is given as follows:

1. $\quad$ grouping the bits of $A$ into $2^{d} 4$-bit elements to obtain $Q_{0}$;
2. for $r=0$ to $5(d-1)-1, \quad Q_{r+1}=R_{d}\left(Q_{r}, C_{r}^{(d)}\right)$;
3. $\quad Q_{5(d-1)+1}=R_{d}^{*}\left(Q_{5(d-1)}, C_{5(d-1)}^{(d)}\right)$;
4. de-grouping the $2^{d} 4$-bit elements in $Q_{5(d-1)+1}$ to obtain $B$;

The grouping of bits into 4 -bit elements in the first step and the de-grouping in the last step are designed to achieve efficient bit-slice software implementation. The grouping in the first step is given as follows (as shown in Fig. 6):
for $i=0$ to $2^{d-1}-1$,
\{

$$
q_{0,2 i}=A^{i}\left\|A^{i+2^{d}}\right\| A^{i+2 \cdot 2^{d}} \| A^{i+3 \cdot 2^{d}} ;
$$

$$
q_{0,2 i+1}=A^{i+2^{d-1}}\left\|A^{i+2^{d-1}+2^{d}}\right\| A^{i+2^{d-1}+2 \cdot 2^{d}} \| A^{i+2^{d-1}+3 \cdot 2^{d}} ;
$$

\}


Figure 6: The grouping in function $E_{d}$

The de-grouping in the last step is given as follows (as shown in Fig. 7):
for $i=0$ to $2^{d-1}-1$, \{

$$
B^{i}\left\|B^{i+2^{d}}\right\| B^{i+2 \cdot 2^{d}} \| B^{i+3 \cdot 2^{d}}=q_{5(d-1)+1,2 i} ;
$$

$$
B^{i+2^{d-1}}\left\|B^{i+2^{d-1}+2^{d}}\right\| B^{i+2^{d-1}+2 \cdot 2^{d}} \| B^{i+2^{d-1}+3 \cdot 2^{d}}=q_{5(d-1)+1,2 i+1} ;
$$

\}


Figure 7: The de-grouping in function $E_{d}$

The round constants of $E_{d}$ are given in Sect. 4.6.

### 4.6 Round constants of $E_{d}$

The round constants $C_{r}^{(d)}$ for $E_{d}$ are generated from the round function $R_{d-2}$ (with all the round constants of $R_{d-2}$ being set as 0 ). Each $C_{r}^{(d)}$ is a $2^{d}$-bit word. They are generated as follows:

$$
\begin{aligned}
& C_{0}^{(d)} \text { is the integer part of }(\sqrt{2}-1) \times 2^{2^{d}} \text { (in big endian form) ; } \\
& C_{r}^{(d)}=R_{d-2}\left(C_{r-1}^{(d)}, 0\right) \text { for } 1 \leq r \leq 5(d-1) .
\end{aligned}
$$

The values of $C_{r}^{(8)}(0 \leq r \leq 35)$ are given in Appendix A.1.

## 5 Compression Function $F_{d}$

Compression function $F_{d}$ is constructed from the function $E_{d} . F_{d}$ compresses the $2^{d+1}$-bit message block $M^{(i)}$ and $2^{d+2}$-bit $H^{(i-1)}$ into the $2^{d+1}$-bit $H^{(i)}$ :

$$
H^{(i)}=F_{d}\left(H^{(i-1)}, M^{(i)}\right) .
$$

The construction of $F_{d}$ is shown in Fig. 8. According to the definition of $E_{d}$, the input to every first-layer Sbox would be affected by two message bits; and the output from every last-layer Sbox would be XORed with two message bits.


Figure 8: The compression function $F_{d}$

## $5.1 \quad F_{8}$

$F_{8}$ is the compression function used in hash function JH. $F_{8}$ compresses the 512 -bit message block $M^{(i)}$ and 1024 -bit $H^{(i-1)}$ into the 1024 -bit $H^{(i)} . F_{8}$ is constructed from $E_{8}$. Let $A, B$ denote two 1024 -bit words. The computation of $H^{(i)}=F_{8}\left(H^{(i-1)}, M^{(i)}\right)$ is given as:

1. $A^{j}=H^{(i-1), j} \oplus M^{(i), j} \quad$ for $0 \leq j \leq 511$;

$$
A^{j}=H^{(i-1), j} \quad \text { for } 512 \leq j \leq 1023 ;
$$

2. $\quad B=E_{8}(A)$;
3. $H^{(i), j}=B^{j} \quad$ for $0 \leq j \leq 511$;

$$
H^{(i), j}=B^{j} \oplus M^{(i), j-512} \quad \text { for } 512 \leq j \leq 1023 ;
$$

## 6 JH Hash Algorithms

Hash function JH consists of five steps: padding the message $M$ (Sect. 6.1), parsing the padded message into message blocks (Sect. 6.2), setting the initial hash value $H^{(0)}$ (Sect. 6.3), computing the final hash value $H^{(N)}$ (Sect. 6.4), and generating the message digest by truncating $H^{(N)}$ (Sect. 6.5).

### 6.1 Padding the message

The message $M$ is padded to be a multiple of 512 bits. Suppose that the length of the message $M$ is $\ell$ bits. Append the bit " 1 " to the end of the message, followed by $896-1-(\ell \bmod 512)$ zero bits, then append the 128 -bit block that is equal to the number $\ell$ expressed using a binary representation in big endian form. Thus at least 512 additional bits are padded to the message $M$.

### 6.2 Parsing the padded message

After a message has been padded, it is parsed into $N$ 512-bit blocks, $M^{(1)}$, $M^{(2)}, \ldots, M^{(N)}$. The 512 -bit message block is expressed as four 128 -bit words. The first 128 bits of message block $i$ are denoted as $M_{0}^{(i)}$, the next 128 bits are $M_{1}^{(i)}$, and so on up to $M_{3}^{(i)}$.

### 6.3 Setting the initial hash value $H^{(0)}$

The initial hash value $H^{(0)}$ is set depending on the message digest size. The first two bytes of $H^{(-1)}$ are set as the message digest size, and the rest bytes of $H^{(-1)}$ are set as 0 . Set $M^{(0)}$ as 0 . Then $H^{(0)}=F_{8}\left(H^{(-1)}, M^{(0)}\right)$.

More specifically, the value of $H_{0}^{(-1), 0}\left\|H_{0}^{(-1), 1}\right\| \cdots \| H_{0}^{(-1), 15}$ is $0 x 00 E 0$, $0 x 0100$, $0 x 0180,0 x 0200$ for JH-224, JH-256, JH-384 and JH-512, respectively. Let $H^{(-1), j}=0$ for $16 \leq j \leq 1023$. Set the 512 -bit $M^{(0)}$ as 0 . The 1024 -bit initial hash value $H^{(0)}$ is computed as

$$
H^{(0)}=F_{8}\left(H^{(-1)}, M^{(0)}\right) .
$$

### 6.4 Computing the final hash value $H^{(N)}$

The compression function $F_{8}$ is applied to generate $H^{(N)}$ by compressing $M^{(1)}, M^{(2)}, \ldots, M^{(N)}$ iteratively. The 1024 -bit final hash value $H^{(N)}$ is computed as follows:

$$
\begin{aligned}
& \text { for } i=1 \text { to } N \\
& \qquad H^{(i)}=F_{8}\left(H^{(i-1)}, M^{(i)}\right)
\end{aligned}
$$

### 6.5 Generating the message digest

The message digest is generated by truncating $H^{(N)}$.

### 6.5.1 JH-224

The last 224 bits of $H^{(N)}$ are given as the message digest of JH-256:

$$
H^{(N), 800}\left\|H^{(N), 769}\right\| \cdots \| H^{(N), 1023} .
$$

### 6.5.2 JH-256

The last 256 bits of $H^{(N)}$ are given as the message digest of JH-256:

$$
H^{(N), 768}\left\|H^{(N), 769}\right\| \cdots \| H^{(N), 1023} .
$$

### 6.5.3 JH-384

The last 384 bits of $H^{(N)}$ are given as the message digest of JH-384:

$$
H^{(N), 640}\left\|H^{(N), 641}\right\| \cdots \| H^{(N), 1023} .
$$

### 6.5.4 JH-512

The last 512 bits of $H^{(N)}$ are given as the message digest of JH-512:

$$
H^{(N), 512}\left\|H^{(N), 513}\right\| \cdots \| H^{(N), 1023} .
$$

## 7 Bit-Slice Implementation of JH

The description of JH given in Sect. 4 and Sect. 5 are suitable for efficient hardware implementation. In this section, we illustrate the bit-slice implementation of JH. The bit-slice implementation of $F_{d}$ uses $d-1$ different round function descriptions (the hardware description of $F_{d}$ uses identical round function description).

### 7.1 Bit-slice parameters

The following additional parameters are used in the bit-slice implementation of JH.
$C_{r}^{\prime(d)} \quad$ The round constant words used in the bit-slice implementation of $E_{d}$ with $0 \leq r \leq 5 \times(d-1)$. Each $C_{r}^{\prime(d)}$ is a $2^{d}$-bit constant word.
$C_{r, \text { even }}^{\prime(d)} \quad$ Even bits of $C_{r}^{\prime(d)} . C_{r, \text { even }}^{\prime(d)}=C_{r}^{\prime(d), 0}\left\|C_{r}^{\prime(d), 2}\right\| C_{r}^{\prime(d), 4} \|$ $\cdots \| C_{r}^{\prime(d), 2^{d}-2}$. Each $C_{r, \text { even }}^{\prime(d)}$ is a $2^{d-1}$-bit constant word. $C_{r, o d d}^{\prime(d)} \quad$ Odd bits of $C_{r}^{\prime(d)} . C_{r, o d d}^{(d)}=C_{r}^{\prime(d), 1}\left\|C_{r}^{\prime(d), 3}\right\| C_{r}^{\prime(d), 5} \|$ $\cdots \| C_{r}^{\prime(d), 2^{d}-1}$. Each $C_{r, o d d}^{\prime(d)}$ is a $2^{d-1}$-bit constant word.
$H_{j}^{(i)} \quad$ The $j^{\text {th }} 128$-bit word of the $i^{\text {th }}$ hash value. $H_{0}^{(i)}$ is the left-most 128 -bit word of hash value $H^{(i)}$.
$M_{j}^{(i)} \quad$ The $j^{\text {th }} 128$-bit word of the $i^{\text {th }}$ message block. $M_{0}^{(i)}$ is the left-most word of message block $M^{(i)}$.

### 7.2 Bit-slice functions

The following functions are used in the bit-slice implementation of JH.

### 7.2.1 Sbox

$S^{\text {bitsli }}$ implements both $S_{0}$ and $S_{1}$ in the bit-slice implementation of JH. Let each $x_{i}(0 \leq i \leq 3)$ denotes a $2^{d-1}$-bit word. Let $c$ denote a $2^{d-1}$ bit constant word. let $t$ denote a temporary word. $\left(x_{0}, x_{1}, x_{2}, x_{3}\right)=$ $S^{b i t s l i}\left(x_{0}, x_{1}, x_{2}, x_{3}, c\right)$ is computed in the following 11 steps:

1. $x_{3}=\neg x_{3}$;
2. $\quad x_{0}=x_{0} \oplus\left(c \&\left(\neg x_{2}\right)\right)$;
3. $t=c \oplus\left(x_{0} \& x_{1}\right)$;
4. $x_{0}=x_{0} \oplus\left(x_{2} \& x_{3}\right)$;
5. $\quad x_{3}=x_{3} \oplus\left(\left(\neg x_{1}\right) \& x_{2}\right)$;
6. $\quad x_{1}=x_{1} \oplus\left(x_{0} \& x_{2}\right)$;
7. $x_{2}=x_{2} \oplus\left(x_{0} \&\left(\neg x_{3}\right)\right)$;
8. $x_{0}=x_{0} \oplus\left(x_{1} \mid x_{3}\right)$;
9. $x_{3}=x_{3} \oplus\left(x_{1} \& x_{2}\right)$;
10. $x_{1}=x_{1} \oplus\left(t \& x_{0}\right)$;
11. $x_{2}=x_{2} \oplus t$;

### 7.2.2 Linear Transform

$L^{\text {bitsli }}$ implements the linear transform in the bit-slice implementation of JH. Let each $a_{i}$ and $b_{i}(0 \leq i \leq 7)$ denotes a $2^{d-1}$-bit word. $\left(b_{0}, b_{1}, \cdots, b_{7}\right)=$ $L^{\text {bitsli }}\left(a_{0}, a_{1}, \cdots, a_{7}\right)$ is computed as follows:

$$
\begin{array}{ll}
b_{4}=a_{4} \oplus a_{1} ; \quad b_{5}=a_{5} \oplus a_{2} ; \\
b_{6}=a_{6} \oplus a_{3} \oplus a_{0} ; b_{7}=a_{7} \oplus a_{0} ; \\
b_{0}=a_{0} \oplus b_{5} ; \quad b_{1}=a_{1} \oplus b_{6} ; \\
b_{2}=a_{2} \oplus b_{7} \oplus b_{4} ; b_{3}=a_{3} \oplus b_{4} .
\end{array}
$$

### 7.2.3 Permutation $\bar{\omega}$

Let $A=\left(a_{0}, a_{1}, \cdots, a_{2 \times \alpha \times n-1}\right)$, where $\alpha$ and $n$ are positive integers. Let $B=\left(b_{0}, b_{1}, \cdots, b_{2 \times \alpha \times n-1}\right)$. Each $a_{i}$ and $b_{i}$ denotes a 4 -bit element. The permutation $B=\bar{\omega}(A, n)$ is computed as follows:

$$
\begin{aligned}
& \text { for } i=0 \text { to } \alpha-1, \\
& \quad \text { for } j=0 \text { to } n-1, \\
& \quad b_{2 \times i \times n+j}=a_{2 \times i \times n+n+j} ; b_{2 \times i \times n+n+j}=a_{2 \times i \times n+j} ;
\end{aligned}
$$

For example, $\bar{\omega}(A, 1)$ swaps element $a_{2 i}$ and $a_{2 i+1}$.

### 7.2.4 Permutation $\omega$

Permutation $\omega(A, n)$ swaps the bits in a word $A$. It is computed by treating each bit in $A$ as an element, then applying the permutation $\bar{\omega}(A, n)$.

### 7.2.5 Permutation $\bar{\sigma}_{d}$

Permutation $\bar{\sigma}_{d}$ operates on $2^{d}$ elements. Let $A=\left(a_{0}\left\|a_{1}\right\| \cdots \| a_{2^{d}-1}\right)$, $B=\left(b_{0}\left\|b_{1}\right\| \cdots \| b_{2^{d}-1}\right)$. Let $n=2^{\beta}$, where $\beta$ is an integer smaller than $d-1$. $B=\bar{\sigma}_{d}(A, n)$ permutes the odd elements in $A$ as follows:

$$
\begin{aligned}
\left(b_{1}, b_{3}, b_{5}, \cdots, b_{2^{d}-1}\right) & =\bar{\omega}\left(\left(a_{1}, a_{3}, a_{5}, \cdots, a_{2^{d}-1}\right), n\right) ; \\
\left(b_{0}, b_{2}, b_{4}, \cdots, b_{2^{d}-2}\right) & =\left(a_{0}, a_{2}, a_{4}, \cdots, a_{2^{d}-2}\right) .
\end{aligned}
$$

### 7.2.6 Permutation $\sigma_{d}$

Permutation $\sigma_{d}(A, n)$ operates on the bits in a word $A$. It is computed by treating each bit in $A$ as an element, then applying the permutation $\overline{\sigma_{d}}(A, n)$.

### 7.2.7 Round constants

Let $I P_{d}$ denote the inverse of $P_{d}$. Let $I P_{d}^{r}$ denote the composition of $r$ permutation $I P_{d}$ :

$$
I P_{d}^{r}=\underbrace{I P_{d} \circ I P_{d} \circ \cdots \circ I P_{d}}_{r} .
$$

Note that $I P_{d}^{r}$ has the property that $I P_{d}^{r}=I P_{d}^{r+\alpha \cdot d}$.
Let permutation $\lambda_{d}^{r}(A)$ operate on the bits in a word $A$. It is computed by treating each bit in $A$ as an element, then applying the permutation $I P_{d}^{r}$.

Let $\eta_{d}^{r}$ denote a permutation. Let $A, B$ and $V_{i}$ denote $2^{d}$-bit words. $B=\eta_{d}^{r}(A)$ is computed as follows:

$$
\begin{aligned}
& V_{0}=A ; \\
& \text { for } i=0 \text { to } r-1, \quad V_{i+1}=\sigma_{d}\left(V_{i}, 2^{i \bmod (d-1)}\right) ; \\
& B=V_{r} ;
\end{aligned}
$$

The round constant $C_{r}^{\prime(d)}$ is generated from $C_{r}^{(d)}$ as:

$$
C_{r}^{\prime(d)}=\eta_{d}^{r} \circ \lambda_{d}^{r}\left(C_{r}^{(d)}\right) .
$$

The $2^{d-1}$-bit constant words $C_{r, \text { even }}^{\prime(d)}$ and $C_{r, o d d}^{\prime(d)}$ are obtained by extracting the even and odd bits of $C_{r}^{\prime(d)}$, respectively, as defined in Sec. 7.1. $C_{r, \text { even }}^{\prime(8)}$ and $C_{r, o d d}^{\prime(8)}$ are given in Appendix A.2.

### 7.2.8 An alternative description of round function $R_{d}$

The description of $R_{d}$ in Sect. 4.4 is suitable for hardware implementation. But that description is not suitable for the bit-slice implementation. We give here an alternative description of $R_{d}$, and denote the $r$-th round function as $R_{d, r}^{\prime}$. The $2^{d}$-bit round constant of the $r$-th round is denoted as $C_{r}^{\prime(d)}$. Let $V=v_{0}\left\|v_{1}\right\| \cdots \| v_{2^{d}-1}$, where each $v_{i}$ denotes a 4 -bit word. The computation of $B=R_{d, r}^{\prime}\left(A, C_{r}^{\prime(d)}\right)$ is given as follows:

1. for $i=0$ to $2^{d}-1$,
\{

$$
\text { if } C_{r}^{\prime(d), i}=0 \text {, then } v_{i}=S_{0}\left(a_{i}\right) \text {; }
$$

$$
\text { if } C_{r}^{\prime(d), i}=1 \text {, then } v_{i}=S_{1}\left(a_{i}\right) \text {; }
$$

\}
2. $B=\bar{\sigma}_{d}\left(V, 2^{r \bmod (d-1)}\right)$

Note that $R_{d, r}^{\prime}$ has the following properties:

1. The description of $R_{d, r}^{\prime}$ is the same as $R_{d, r+\alpha \cdot(d-1)}^{\prime}$ except for the different round constants.
2. For the same input passing through multiple rounds, at the end of the $\alpha \cdot(d-1)$-th round, the output from $R_{d, \alpha \cdot(d-1)}^{\prime}$ is identical to the output from $R_{d, \alpha \cdot(d-1)}$.
Six rounds of $R_{4, r}^{\prime}(0 \leq r \leq 5)$ are illustrated in Fig. 9.

### 7.2.9 Bit-slice implementation of round function $R_{d}$

The above description of $R_{d, r}^{\prime}$ can be implemented efficiently in a bit-slice way. The method used is to separate the odd and even elements of $A$ in $R_{d, r}^{\prime}$. Denote the bit-slice implementation as $R_{d, r}^{\text {bitsli }}$. Let $A$ and $B$ represent two $2^{d+2}$-bit words, $A=a_{0}\left\|a_{1}\right\| a_{2}\|\cdots\| a_{7}$, and $B=b_{0}\left\|b_{1}\right\| b_{2}\|\cdots\| b_{7}$, where each $A_{i}$ and $B_{i}$ represents a $2^{d-1}$-bit word. Let each $v_{i}$ and $u_{i}(0 \leq i \leq$ 7) denote a $2^{d-1}$-bit word. The computation of $B=R_{d, r}^{\text {bitsli }}\left(A, C_{r, e v e n}^{\prime(d)}, C_{r, o d d}^{\prime(d)}\right)$ is given as follows:

1. $\left(v_{0}, v_{2}, v_{4}, v_{6}\right)=S^{\text {bitsli }}\left(a_{0}, a_{2}, a_{4}, a_{6}, C_{r, \text { even }}^{\prime(d)}\right)$;

$$
\left(v_{1}, v_{3}, v_{5}, v_{7}\right)=S^{b i t s l i}\left(a_{1}, a_{3}, a_{5}, a_{7}, C_{r, o d d}^{\prime(d)}\right)
$$

2. $\quad\left(u_{0}, u_{2}, u_{4}, u_{6}, u_{1}, u_{3}, u_{5}, u_{7}\right)=L^{\text {bitsli }}\left(v_{0}, v_{2}, v_{4}, v_{6}, v_{1}, v_{3}, v_{5}, v_{7}\right)$;
3. $b_{0}=u_{0} ; b_{2}=u_{2} ; b_{4}=u_{4} ; b_{6}=u_{6} ;$
$b_{1}=\omega\left(u_{1}, 2^{r \bmod (d-1)}\right)$;
$b_{3}=\omega\left(u_{3}, 2^{r \bmod (d-1)}\right) ;$
$b_{5}=\omega\left(u_{5}, 2^{r \bmod (d-1)}\right) ;$
$b_{7}=\omega\left(u_{7}, 2^{r \bmod (d-1)}\right) ;$

### 7.2.10 Bit-slice implementation of $E_{d}$

The $2^{d+2}$-bit input and output are denoted as $A$ and $B$, respectively. Let each $Q_{r}$ denote a $2^{d+2}$-bit word for $0 \leq r \leq 5(d-1)$. Let $R_{d, r}^{* b i t s l i}$ denote the round function $R_{d, r}^{* b i t s l i}$ with only the Sbox layer. The computation of $B=E_{d}(A)$ is given as follows:

1. $Q_{0}=A$;
2. for $r=0$ to $5(d-1)-1, \quad Q_{r+1}=R_{d, r}^{\text {bitsli }}\left(Q_{r}, C_{r, e v e n}^{\prime(d)}, C_{r, o d d}^{\prime(d)}\right)$;
3. $\quad B=R_{d, 5(d-1)}^{* b i t s l i}\left(Q_{5(d-1)}, C_{5(d-1), \text { even }}^{\prime(d)}, C_{5(d-1), \text { odd }}^{\prime(d)}\right)$;

The generation of the round constants is given in Sect. 7.2.7.


Figure 9: An alternative description of 6 rounds of $R_{4}$ (constant bits not shown)

### 7.3 Bit-slice implementation of $F_{8}$

$F_{8}$ compresses the 512 -bit message block $M^{(i)}$ and 1024 -bit $H^{(i-1)}$ into the 1024-bit $H^{(i)}$. The computation of $H^{(i)}=F_{8}\left(H^{(i-1)}, M^{(i)}\right)$ is given as:

1. $\quad A_{j}=H_{j}^{(i-1)} \oplus M_{j}^{(i)} \quad$ for $0 \leq j \leq 3$;

$$
A_{j}=H_{j}^{(i-1)} \quad \text { for } 4 \leq j \leq 7
$$

2. $\quad B=E_{8}(A)$;
3. $H_{j}^{(i)}=B_{j} \quad$ for $0 \leq j \leq 3$;

$$
H_{j}^{(i)}=B_{j} \oplus M_{j-4}^{(i)} \quad \text { for } 4 \leq j \leq 7
$$

Note that in round function $R_{8, r}^{b i t s l i}\left(A, C_{r, \text { even }}^{\prime(8)}, C_{r, o d d}^{\prime(8)}\right)$, each word is 128-bit and is thus suitable for SSE2 implementation. For a 128 -bit word $x, \omega(x, n)$ can be implemented with two AND operations (AND with a constant to extract the bits to be swapped), two shift operations and one OR operations (note that the shift operations would be affected by the endianess of the SSE2 register). In addition, $\omega(x, 32)$ and $\omega(x, 64)$ can be implemented with one SSE2 shuffle operation. Thus the SSE2 implementation of $F_{8}$ is very efficient.

## 8 Variants of JH

The design of JH hash algorithms implies several variants by varying the parameter $d$ or by replacing $P_{d}$ with $P_{d}^{\prime}$ in round function $R_{d}$.

### 8.1 Varying the parameter $d$

The compression function $F_{d}$ gives several compression functions by varying the parameter $d$.
$F_{6}$. $d=6$. The round number is increased from $25(=5(d-1))$ to 30 (=6(d-1)). With 256 -bit block size and 128 -bit message block size, this compression function is extremely hardware efficient. Hash function using this compression function can achieve 128 -bit security level for collision resistance, preimage resistance and second preimage resistance for 256 -bit message digest size.
$F_{7} . d=7$. With 512 -bit block size and 256 -bit message block size, this compression is used to generate 256 -bit message digest size. The memory required is half of that of $F_{8}$, and it achieves 128-bit security level for comllision resistance, 256 -bit security for preimage resistance.
$F_{9} . d=9$. With 2048-bit block size, this compression function is extremely efficient on the future microprocessors that support shift and binary operations over 256 -bit registers.

### 8.2 Replacing $P_{d}$ with $P_{d}^{\prime}$

Replacing permutation $P_{d}$ with $P_{d}^{\prime}$ in round function $R_{d}$, and change the round number $5(d-1)$ to $5 d$ in $E_{d}$, we can obtain another family of com-
pression functions. This family of compression functions are slightly simpler in hardware, but its bit-slice implementation requires twice amount of shift operations as required in $F_{d}$. A few variants can be obtained by varying the value of $d$.

## 9 Security Analysis of JH

The security of JH hash algorithms are stated below ( $\bar{l}$ denotes the number of message blocks, the length of a message is less than $2^{128}$ bits):

|  | collision | second- <br> preimage | preimage |
| :--- | :--- | :--- | :--- |
| JH-224 | $2^{112}$ | $2^{224}$ | $2^{224}$ |
| JH-256 | $2^{128}$ | $2^{256}$ | $2^{256}$ |
| JH-384 | $2^{192}$ | $2^{384}$ | $2^{384}$ |
| JH-512 | $2^{256}$ | $2^{512-\bar{l}}$ | $2^{512}$ |

Note that the second-preimage resistance of JH-512 is affected by herding attack [10]. The reason is that the collision resistance of Jh-512 is $2^{256}$, although the size of the hash value $H^{(i)}$ is 1024 bits. However, the secondpreimage resistance of JH-512 would not be affected by herding attack if birthday attack is applied to find collisions in herding attack.

### 9.1 Differential cryptanalysis

Differential cryptanalysis is important in analyzing the security of a hash function. It has been applied to break MD4, MD5, SHA-0 and SHA-1 $[8,5,2,3,14,15,16,17]$.

We study the number of active Sboxes being involved in a differential characteristic in $E_{d}$. The symmetry structure of EDP design allows us to determine the number of active Sboxes easily since many differential paths (branches) are equivalent. We can replace $P_{d}$ with $P_{d}^{\prime}$ in $E_{d}$ to get a simpler variant whose security is equivent to that of the original $E_{d}$. We can also study the $E_{d}$ with small $d$ to learn when the minimum number of active Sboxes would occur. For example, two active Sboxes before the linear transformation $L$ would result in only one active Sboxes after $L$.

For $d \in 2,3,4$, we exhaustively searched for the minimum number of active Sboxes. The minimum number of active Sboxes for $2 d+1$ Sbox layers is $10,20,38$ for $d=2,3,4$, respectively. For $d \geq 4$, we found that the minimum number of active Sboxes for $2 d+1$ Sbox layers is $64,112,176,296$ for $d=5,6,7,8$. It shows that the minimum number of active Sboxes does increase significantly as the value of $d$ increases.

For $E_{8}$, we found that the minimum number of active Sboxes for 36 Sbox layers is 624 when there are eight active elements in the input of $E_{8}$. If we
conservatively assume that there are $2^{36}$ multiple paths for a differential, there are still around 600 effective Sboxes. The large number of active Sboxes shows that JH is strong against the differential cryptanalysis.

### 9.1.1 Effect of correlated active elements in differential attack

In the differential cryptanalysis of JH, each differential characteristic of an Sbox has a probability of at most $\frac{1}{4}$. Each active Sbox may contribute $2^{-2}$ to the overall differential probability if the active SBoxes are assumed to be independent. However, when there is correlation between active elements, the overall differential probability may increase.

For the 8 -bit-to-8-bit super Sbox (concept from Rijmen and Daemen) consisting of two nonlinear layers ( 4 Sboxes connected by $L$ ), a differential characteristic has a maximum probability of $\frac{12}{256}=2^{-4.41}$. If we consider that there are 16 combinations of those 4 Sboxes, then the average of those 16 maximum differential probabilities is $\frac{10.875}{256}=2^{-4.56}$. If only 3 Sboxes are active, then the maximum differential probability is $\frac{10}{256}=2^{-4.68}$. For the 16 -bit-to- 16 -bit super Sbox consisting of three nonlinear layers, there are 4096 combinations of those 12 Sboxes. If there is only one active Sbox in the first or last Sbox layer, then there are 7 active Sboxes being involved; the maximum differential probability is $\frac{44}{2^{16}}=2^{-10.54}$, and the average of those 4096 maximum differential probabilities is $2^{-10.98}$. When the minimum number of active Sboxes occurs, we are mainly dealing the 8 -bit-to- 8 -bit super Sbox with 3 active Sboxes, and the 16 -bit-to- 16 -bit super Sbox with 7 active Sboxes. In these situations, we see that the effective differential characteristic of an active Sbox is less than $2^{-1.5}$ (but larger than $2^{-2}$ ).

If we consider that each active Sbox contributes $2^{-1.5}$ to the overall differential probability, then the probability of a differential involves 600 active Sboxes is about $2^{-900}$.

### 9.1.2 Effect of message modification in differential attack

To study the collision resistance of JH , we conservatively assume that an attacker can efficiently eliminate 16 rounds of $E_{8}$ with message modification, then there are 20 Sbox layers being left. For 20 Sbox layers of $E_{8}$, we found that a differential characteristic involves at least 336 active Sboxes. If we assume that there are $2^{20}$ multiple paths for a differential, then a differential has probability less than $2^{-1.5 \times 336} \times 2^{20}=2^{-484}$. We thus expect a differential collision attack can not succeed with less than $2^{256}$ operations.

### 9.1.3 Second-preimage and preimage differential attacks

For the second-preimage resistance of JH, we note that a differential passing through at least two compression functions $F_{8}$ should be considered. The reason is that the 512 -bit message block size is only half that of the 1024 -bit
hash value $H^{(i)}$, and each compression function involves huge number of active Sboxes. Since message modification in second-preimage attack is not as efficient as that in collision attack, and at least two message blocks are involved in a differential second-preimage attack, we expect that a differential exists with probability much less than $2^{-512}$, and JH is secure against the differential second-preimage attack.

For the preimage resistance of JH , we note that a differential passing through at least two compression functions $F_{8}$ should be considered. The reason is that one more block is padded to the message before generating the message digest. We expect that the complexity of the preimage attack is more than the square of that of the collision attack. We thus expect that JH is secure against the differential preimage attack.

### 9.2 Truncated differential cryptanalysis

For collision search, truncated differential cryptanalysis [11] may be viewed as the bridge linking differential cryptanalysis and birthday attack. Differential cryptanalysis can be viewed as truncated differential cryptanalysis with input space 2 , while birthday cryptanalysis can be viewed as truncated differential cryptanalysis with input space $2^{n}$ where $n$ indicates the block size of the compression function. Because of the nature of EDP design, it is necessary to evaluate the security of JH against the truncated differential cryptanalysis.

In the truncated differential cryptanalysis of JH, we focus on whether an element is active or not instead of the value of the difference. Let us consider those four Sboxes connected by a linear transformation $L$. If only one of the two Sboxes before $L$ is active, then both Sboxes after $L$ are active with probability 1 . We call this event as active element expansion. If both two Sboxes before $L$ are active and independent, then the probability that only one Sbox after $L$ is active is $2^{-4}$. We call this event as active element shrinking. If there are independent active Sboxes in the last Sbox layer, then the probability that the difference of the output from an active Sbox is cancelled by the message difference (if there is message difference at that location) is $2^{-4}$. For a truncated differential characteristic, we count the number of active element shrinking events and the number of active Sboxes in the last Sbox layer of $E_{8}$, and denote the sum of these two numbers as $T D_{8}$.

Exploiting the symmetry property of $E_{8}$, we found in our analysis that the smallest value of $T D_{8}$ is 200 when there are eight active elements in the input of $E_{8}$. If we assume that the message modification can effectively remove 8 rounds in the truncated differential attack (the message modification in truncated differential attack is a bit difficult), then the smallest value of $T D_{8}$ is 144 when there are eight active elements in the input of $E_{8}$. Assume that there are $2^{26}$ multiple paths, it requires around $2^{144 \times 4-26}=2^{550}$ differ-
ence pairs to generate a collision. Note that $2^{32}$ messages with eight active elements can generate only $2^{63}$ difference pairs, the attack would require much more than $2^{256}$ messages.

Truncated differential cryptanalysis is not that efficient for preimage and second-preimage attack. We thus do not apply truncated differential cryptanalysis to find the preimage and second-preimage of JH.

### 9.3 Algebraic attacks

Algebraic attacks solve the nonlinear equations in order to recover the key or message. For hash function cryptanalysis, algebraic attacks can be applied to find collision, second preimage and preimage if the algebraic equations of the compression function are very weak.

In the past several years, algebraic attacks have been proposed against block ciphers, but so far there is no evidence that algebraic attacks can break a practical block cipher faster than statistical cryptanalysis techniques, and there is no evidence that the complexity of algebraic attacks against block ciphers would be linear to the round number. The recent cube attacks, developed by Dinur and Shamir [7], can solve nonlinear equations with low degree when a number of equations (involve the same secret key) are available.

To find a collision of JH hash algorithms with algebraic attack, the meet-in-the-middle approach can result in algebraic equations of 18 Sbox layers. To find a second-preimage with algebraic attack, two blocks of message must be considered, and thus an algebraic attack needs to deal with algebraic equations of 36 Sbox layers. Recovering a message from the message digest would involve at least 36 Sbox layers since one more block is padded to the message. Because of the algebraic degree of the Sbox is 3 and the number of rounds being involved is large, we consider that JH is secure against algebraic attacks.

To be conservative, we use constant bits to select Sboxes to further strengthen JH against algebraic attacks.

## 10 Performance of JH

JH can be implemented efficiently on a wide range of platforms ranging from one-bit processor (hardware) to 128 -bit processor (SIMD/SSE2). The reason is that EDP design allows JH being constructed from extremely simple elements. The 5-bit-to-4-bit (including the constant bit) Sbox can be implemented with 20 binary operations (including ANDNOT operation), and the linear transformation $L$ can be implemented with 10 binary operations. The simple Sboxes and linear transformation ensures that JH is extremely hardware and software efficient.

### 10.1 Hardware

The hardware implementation of JH is extremely simple and efficient due to the simple Sboxes and linear transformation. JH uses 1024-bit memory for storing the state of $E_{8}, 512$-bit memory for storing the memory, and 256 -bit memory to store a round constant (if the round constants are generated on-the-fly).

Let us compare JH with the ultra-lightweight block cipher PRESENT [4]. The hardware complexity of JH is comparable to that of PRESENT, except for the difference in block sizes. JH uses slightly more complicated Sboxes and linear transformation than PRESENT. The block size of $E_{8}$ is about 16 times that of PRESENT, while the size of a round constant in $E_{8}$ is only 4 times that of key size of PRESENT. A rough estimation is that $E_{8}$ requires 16 times more gates than PRESENT. PRESENT uses about 1570 GE (gate equivalents), so JH may require $1570 \times 16 \approx 25 \mathrm{~K}$ GE (estimated).

### 10.2 8-bit processor

JH can be implemented on 8-bit processor in two approaches. One approach is to implement the hardware description of JH with table lookup for Sboxes. The advantage of this approach is that the constant bits can be generated on-the-fly efficiently. Another approach is to implement the bit-slice description of JH. With 1152-byte precomputed round constants being stored in ROM, this implementation is expected to be quite fast. Given that the SSE2 bitslice implementation of JH runs at 16.8 cycles/byte on CORE 2 processor, we can roughly estimate the speed of JH on 8-bit processor. The register size of 8 -bit processor is 16 times smaller than that of SSE2 register. If we estimate that the number of instructions being processed per clock cycle on 8 -bit processor is 5 times less than that on CORE 2 processor, the speed of the bit-slice implementation of JH on 8 -bit processor is about $16 \times 5 \times 16.8=1344$ cycles/byte (estimated).

### 10.3 Core 2 processor

The bit-slice implementation of JH is tested on the popular Core 2 processor. The processor being used in the test is Core 2 Duo Mobile Processor P9400 2.53 GHz . The Operating systems are 32 -bit and 64 -bit Windows Vista Business. The compiler being used is the Intel C compiler 10.1.025 (IA32 version of the compiler is used with the 32 -bit Vista, and Intel-64 version of the compiler is used with the 64 -bit Vista). The hash speed (for long message) is 16.8 clock cycles/byte with the 64 -bit Vista (with optimization option -QxT -O2 of the Intel-64 Intel C compiler); and it is 21.3 clock cycles/byte on the 32 -bit Vista (with optimization option -QxT of the IA32 Intel C compiler).

JH on 64 -bit platform is faster than that on 32 -bit platform. The reason is that there are sixteen 128 -bit registers on the 64 -bit platform of Core 2 processor; while there are only eight 128 -bit registers on the 32 -bit platform of Core 2 processor.

Microsoft Visual C++ 2005 and 2008 are not recommended for compiling the SSE2 codes. It seems that the optimization of SSE2 instructions is not implemented (or very poor) in Microsoft Visual C++ 2005 and 2008. The speed of JH is about $40+$ clock cycles/byte with the Microsoft compilers (with optimization option /O2).

## 11 Design Rationale

We give below the rationale of designing the components of JH.

### 11.1 Compression function $F_{d}$

The construction of compression function $F_{d}$ from bijective function $E_{d}$ is new. It gives an extremely simple and efficient approach to construct a compression function from a bijective function.

In $F_{d}$, the message block size is half of the block size of $E_{d}$. The message is XORed with the first half of the input to $E_{d}$, then it is XORed with the second half of the output from $E_{d}$ to achieve one-wayness. Besides the onewayness, this construction is very efficient - every bit in the output from $E_{d}$ is not truncated; and the difference cancellation involving the message is minimized. The message block size is only half of the block size of $E_{d}$, it is to prevent copying a collision block to other locations, and it is also helpful to resist attacks launched from the middle of $E_{d}$.

In the hash function, at least one more block is appended to the message. The reason is that if the difference of two last message blocks eliminates the difference of the inputs to $E_{d}$, then the outputs from $F_{d}$ are not random. Thus one more $F_{d}$ operation is needed to randomize the hash value.

### 11.2 EDP design

EDP design (Sec. 2) being used to construct the bijective function $E_{d}$ is very simply and efficient. The input to $E_{d}$ is grouped into a $2^{d}$-dimensional array. The nonlinear layer consists of Sboxes. In the linear layer of the $r$-th round, MDS code is applied along the $(r \bmod d)$-th dimension of the array. EDP design is the generalization of the AES design [6].

EDP design is easy to analyze due to its symmetrical construction. Round constants are applied to prevent the symmetry property being exploited in attacks.

EDP design is efficient in hardware since $E_{d}$ can be build upon small components. EDP design is also efficient in software since it can be implemented in a bit-slice approach.

### 11.3 Round number

The round number of $E_{8}$ is $5(8-1)=35$. The round number is chosen to satisfy two requirements. One requirement is that the round number is the multiple of (d-1) so that the hardware description is simple since at the end of the multiple of ( $\mathrm{d}-1$ ) rounds, the output from the hardware description is identical to that from the bit-slice implementation. Another requirement is that the round number should be larger than $4 d$ in order to satisfy the security requirements. We thus set the round number of $E_{8}$ as 35 .

The round number 35 is used for all the JH algorithms for two reasons - one reason is to achieve the simplicity of description and implementation; another reason is to achieve extremely high security for JH-256 (JH-224) so that it achieves 256 -bit (224-bit) security level for preimage and second preimage resistance, and it also eliminates the threat of multicollision attack against JH-224 and JH-256.

### 11.4 Selecting SBoxes

Two Sboxes are used in JH. Each round constant bit selects which Sboxes are used. Similar design has been used in Feistel's block cipher Lucifer [9] in which a key bit selects which Sboxes are used. The main reason that we use two different Sboxes selected by round constant bits is to increase the complexity of the system algebraic equations so that JH can have better resistance against the future algebraic attack.

### 11.5 Designing SBoxes

The 4-bit-to-4-bit Sboxes in JH are designed to meet the following requirements:

1. There is no fixed point for each Sbox, i.e., the input is always different from the output. For the same input, the outputs from two Sboxes are different.
2. Each differential characteristic has a probability of at most $\frac{1}{4}$.
3. Each linear characteristic [12] has a probability in the range $\frac{1}{2} \pm \frac{1}{4}$.
4. The nonlinear order of each output bit as a function of the input bits is 3 .
5. The algebraic normal forms of the two Sboxes are different.
6. The resulting super Sboxes (formed with more than than one Sbox layer, introduced by Rijmen and Daemen, mainly to address the effect of correlated active elements) are strong against differential cryptanalysis.

Putting two Sboxes together, we have a 5 -bit-to-4-bit Sbox with one input bit being the round constant bit that selects which Sboxes are used. This Sbox satisfies the following requirements:

1. Each differential characteristic has a probability of at most $\frac{1}{4}$.
2. Each linear characteristic has a probability in the range $\frac{1}{2} \pm \frac{1}{4}$.

We searched for the 5 -bit-to-4-bit Sbox that can be implemented with small number of operations. The 5 -bit-to-4-bit Sbox being used in JH can be implemented with 20 binary operations (AND, ANDNOT, XOR, NOT, OR).

### 11.6 Linear transform

The linear transform $L$ is probably the simplest $(4,2,3)$ MDS code over $G F\left(2^{4}\right)$. It requires only ten XOR operations.

## 12 Advantages and Limitations

JH hash algorithms have the following advantages:

1. Simple design. Both the hardware and software (bit-slice) descriptions of $F_{8}$ are very simple, easy to implement (however, although both the hardware and software descriptions of JH are simple, it requires some efforts to work out the relations between them).
2. The design of the compression $F_{d}$ gives a simple and efficient way to construct a compression function from a bijective function.
3. EDP design gives a generalized design method of AES.
4. Security analysis can be performed relatively easily. Three approaches are used to achieve this goal. The first approach is to avoid introducing variables into the middle of the compression function so that the differential propagation can be analyzed relatively easily. The second approach is to use the simple EDP design that can greatly simplify the differential cryptanalysis. The third approach is that the EDP involves multidimensional array. The array with low dimension can be easily studied to estimate the strength of the high dimensional array.
5. High efficiency for collision resistance. Three approaches are used. The first approach is to use the EDP design that would likely maximize the difference propagation. The second approach is to minimize the difference cancellation within a compression function. The third approach is to ensure that every operation in a compression function is involved in at least one differential path if there is difference propagation within that compression function.
6. JH can be implemented efficiently over one-bit processor (hardware) to 128 -bit processor (SIMD/SSE2). The reason is that EDP design allows JH being built from extremely simple components.
7. Hardware efficient. The hardware description of JH is simple. The internal state size of $E_{8}$ is only 1024 bits and the message block size is 512 bits. The round constants can be generated on the fly with 256 -bit additional memory. Both the Sboxes and linear transformation in JH are extremely simple.
8. Software efficient. JH is designed to exploit the computational power of modern and widely used microprocessors. The bit-slice description of $E_{8}$ can be efficiently implemented with the SIMD/SSE2 instructions.
9. Several variants are available by varying the parameter $d$. The extremely hardware-efficient $F_{6}$ (with 30 rounds) is suitable for achieving 128 -bit security for collision resistance, preimage resistance and second-preimage resistance.
10. It is convenient to use JH to substitute SHA2 [13] in almost all the SHA2 applications.

## 13 Conclusion

In this document, we proposed JH hash algorithms which are both hardware and software efficient. Our analysis shows that JH is very secure. However, the extensive security analysis of any new design requires a lot of efforts from many researchers. We thus invite and encourage researchers to analyze the security of JH . JH is not covered by any patent and JH is freely-available.

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## References

[1] E. Biham, A. Shamir, "Differential Cryptanalysis of DES-like Cryptosystems." Advances in Cryptology - Crypto'90, LNCS 537, pp. 2-21, Springer-Verlag, 1991.
[2] E. Biham, R. Chen, "Near-Collisions of SHA-0." Advances in Cryptology - CRYPTO 2004, pp. 290-305, Springer-Verlag, 2004.
[3] E. Biham, R. Chen, A. Joux, P. Carribault, C. Lemuet, W. Jalby, "Collisions of SHA-0 and Reduced SHA-1." Advances in Cryptology EUROCRYPT 2005, pp. 36-57, Springer-Verlag, 2005.
[4] A. Bogdanov, L. R. Knudsen, G. Leander, C. Paar, A. Poschmann, M. J. B. Robshaw, Y. Seurin and C. Vikkelsoe, "PRESENT: An UltraLightweight Block Cipher ." Cryptographic Hardware and Embedded Systems - CHES 2007, pp. 450-466, Springer-Verlag, 2007.
[5] F. Chabaud, A. Joux, "Differential Collisions in SHA-0." Advances in Cryptology - CRYPTO 1998, pp. 56-71, Springer-Verlag, 1998.
[6] J. Daeman and V. Rijmen, "AES Proposal: Rijndael." Available on-line from NIST at http://csrc.nist.gov/encryption/aes/rijndael/
[7] I. Dinur and A. Shamir, "Cube Attacks on Tweakable Black Box Polynomials." IACR ePrint, 2008. Available at http://eprint.iacr.org/2008/385
[8] H. Dobbertin, "Cryptanalysis of MD4." Fast Software Encryption FSE 1996, pp. 53-69, Springer-Verlag, 1996.
[9] H. Feistel, "Cryptography and Computer Privacy." Scientific American, vol.228(5), May 1973, pp 15-23.
[10] J. Kelsey, T. Kohno, "Herding Hash Functions and the Nostradamus Attack." Advances in Cryptology - EUROCRYPT 2006, pp. 183-200, Springer-Verlag, 2006.
[11] L. Knudsen, "Truncated and Higher Order Differentials." Fast Software Encryption - FSE94, pp. 196-211, Springer-Verlag.
[12] M. Matsui, "Linear Cryptanalysis Method for DES Cipher." Advances in Cryptology - Eurocrypt'93, LNCS 765, pp. 386-397, Springer-Verlag, 1994.
[13] National Institute of Standards and Technology, "Secure Hash Standard (SHS)." Available at http://csrc.nist.gov/cryptval/shs.html
[14] X. Wang, X. Lai, D. Feng, H. Chen, X. Yu, "Cryptanalysis of the Hash Functions MD4 and RIPEMD." Advances in Cryptology - EU$R O C R Y P T$ 2005, pp. 1-18, Springer-Verlag, 2005.
[15] X. Wang, H. Yu, "How to Break MD5 and Other Hash Functions." Advances in Cryptology - EUROCRYPT 2005, pp. 19-35, SpringerVerlag, 2005.
[16] X. Wang, H. Yu, Y. L. Yin, "Efficient Collision Search Attacks on SHA-0." Advances in Cryptology - CRYPTO 2005, pp. 1-16, SpringerVerlag, 2005.
[17] X. Wang, Y. L. Yin, H. Yu, "Finding Collisions in the Full SHA-1." Advances in Cryptology - CRYPTO 2005, pp. 17-36, Springer-Verlag, 2005.

## A Round constants of $E_{8}$

This section gives the round constants in $E_{8} . E_{8}$ has 36 256-bit round constants.

## A. 1 Round constants in the hardware implementation of $E_{8}$

 The round constants are generated from the first round constant using round function $R_{6}$ (with the round constants of $R_{6}$ being set to 0 ).```
C00 = 6a09e667f3bcc908b2fb1366ea957d3e
    3adec17512775099da2f590b0667322a
C01 = bb896bf05955abcd5281828d66e7d99a
    c4203494f89bf12817deb43288712231
C02 = 1836e76b12d79c55118a1139d2417df5
    2a2021225ff6350063d88e5f1f91631c
C03 = 263085a7000fa9c3317c6ca8ab65f7a7
    713cf4201060ce886af855a90d6a4eed
C04 = 1cebafd51a156aeb62a11fb3be2e14f6
    0b7e48de85814270fd62e97614d7b441
C05 = e5564cb574f7e09c75e2e244929e9549
    279ab224a28e445d57185e7d7a09fdc1
C06 = 5820f0f0d764cff3a5552a5e41a82b9e
    ff6ee0aa615773bb07e8603424c3cf8a
C07 = b126fb741733c5bfcef6f43a62e8e570
    6a26656028aa897ec1ea4616ce8fd510
C08 = dbf0de32bca77254bb4f562581a3bc99
    1cf94f225652c27f14eae958ae6aa616
C09 = e6113be617f45f3de53cff03919a94c3
```

2c927b093ac8f23b47f7189aadb9bc67
C10 = 80d0d26052ca45d593ab5fb310250639 0083afb5ffe107dacfcba7dbe601a12b

C11 = 43af1c76126714dfa950c368787c81ae 3beecf956c85c962086ae16e40ebb0b4
C12 = 9aee8994d2d74a5cdb7b1ef294eed5c1 520724dd8ed58c92d3f0e174b0c32045
C13 = 0b2aa58ceb3bdb9e1eef66b376e0c565 d5d8fe7bacb8da866f859ac521f3d571
C14 = 7a1523ef3d970a3a9b0b4d610e02749d 37b8d57c1885fe4206a7f338e8356866
C15 = 2c2db8f7876685f2cd9a2e0ddb64c9d5 bf13905371fc39e0fa86e1477234a297
$\mathrm{C} 16=9 \mathrm{df} 085 \mathrm{eb} 2544 \mathrm{ebf} 62 \mathrm{~b} 50686 \mathrm{a} 1 \mathrm{e} 6 \mathrm{e} 828$ dfed9dbe0b106c9452ceddff3d138990
C17 = e6e5c42cb2d460c9d6e4791a1681bb2e 222e54558eb78d5244e217d1bfcf5058

C18 = 8f1f57e44e126210f00763ff57da208a 5093b8ff7947534a4c260a17642f72b2

C19 = ae4ef4792ea148608cf116cb2bff66e8 fc74811266cd641112cd17801ed38b59
$\mathrm{C} 20=91 \mathrm{a} 744 \mathrm{efbf68b192d0549b608bdb3191}$ fc12a0e83543cec5f882250b244f78e4
C21 = 4b5d27d3368f9c17d4b2a2b216c7e74e 7714d2cc03e1e44588cd9936de74357c
C22 = Oea17cafb8286131bda9e3757b3610aa 3f77a6d0575053fc926eea7e237df289
C23 = 848af9f57eb1a616e2c342c8cea528b8 a95a5d16d9d87be9bb3784d0c351c32b
$\mathrm{C} 24=\mathrm{c} 0435 \mathrm{cc} 3654 \mathrm{fb} 85 \mathrm{dd} 9335 \mathrm{ba} 91 \mathrm{ac} 3 \mathrm{dbde}$ 1f85d567d7ad16f9de6e009bca3f95b5

C25 = 927547fe5e5e45e2fe99f1651ea1cbf0 97dc3a3d40ddd21cee260543c288ec6b

C26 = c117a3770d3a34469d50dfa7db020300 d306a365374fa828c8b780ee1b9d7a34

C27 = 8ff2178ae2dbe5e872fac789a34bc228 debf54a882743caad14f3a550fdbe68f

C28 = abd06c52ed58ff091205d0f627574c8c bc1fe7cf79210f5a2286f6e23a27efa0
C29 = 631f4acb8d3ca4253e301849f157571d 3211b6c1045347befb7c77df3c6ca7bd
C30 = ae88f2342c23344590be2014fab4f179 fd4bf7c90db14fa4018fcce689d2127b
C31 = 93b89385546d71379fe41c39bc602e8b

7c8b2f78ee914d1f0af0d437a189a8a4

```
C32 = 1d1e036abeef3f44848cd76ef6baa889
```

    fcec56cd7967eb909a464bfc23c72435
    C33 = a8e4ede4c5fe5e88d4fb192e0a0821e9
35ba145bbfc59c2508282755a5df53a5
C34 = 8e4e37a3b970f079ae9d22a499a714c8
75760273f74a9398995d32c05027d810
C35 = 61cfa42792f93b9fde36eb163e978709
fafa7616ec3c7dad0135806c3d91a21b

## A. 2 Round constants in the bit-slice implementation of $E_{8}$

Each round constant used in the bit-slice implementation of $E_{8}$ is linked to the corresponding round constant in the hardware implementation through a permutation.

```
C'00_even = 72d5dea2df15f8677b84150ab7231557
C'00_odd = 81abd6904d5a87f64e9f4fc5c3d12b40
C'01_even = ea983ae05c45fa9c03c5d29966b2999a
C'01_odd = 660296b4f2bb538ab556141a88dba231
C'02_even = 03a35a5c9a190edb403fb20a87c14410
C'02_odd = 1c051980849e951d6f33ebad5ee7cddc
C'03_even = 10ba139202bf6b41dc786515f7bb27d0
C'03_odd = 0a2c813937aa78503f1abfd2410091d3
C'04_even = 422d5a0df6cc7e90dd629f9c92c097ce
C'04_odd = 185ca70bc72b44acd1df65d663c6fc23
C'05_even = 976e6c039ee0b81a2105457e446ceca8
C'05_odd = eef103bb5d8e61fafd9697b294838197
C'06_even = 4a8e8537db03302f2a678d2dfb9f6a95
C'06_odd = 8afe7381f8b8696c8ac77246c07f4214
C'07_even = c5f4158fbdc75ec475446fa78f11bb80
C'07_odd = 52de75b7aee488bc82b8001e98a6a3f4
C'08_even = 8ef48f33a9a36315aa5f5624d5b7f989
C'08_odd = b6f1ed207c5ae0fd36cae95a06422c36
C'09_even = ce2935434efe983d533af974739a4ba7
C'09_odd = d0f51f596f4e81860e9dad81afd85a9f
C'10_even = a7050667ee34626a8b0b28be6eb91727
C'10_odd = 47740726c680103fe0a07e6fc67e487b
C'11_even = 0d550aa54af8a4c091e3e79f978ef19e
C'11_odd = 8676728150608dd47e9e5a41f3e5b062
C'12_even = fc9f1fec4054207ae3e41a00cef4c984
C'12_odd = 4fd794f59dfa95d8552e7e1124c354a5
C'13_even = 5bdf7228bdfe6e2878f57fe20fa5c4b2
C'13_odd = 05897cefee49d32e447e9385eb28597f
```

```
C'14_even = 705f6937b324314a5e8628f11dd6e465
C'14_odd = c71b770451b920e774fe43e823d4878a
C'15_even = 7d29e8a3927694f2ddcb7a099b30d9c1
C'15_odd = 1d1b30fb5bdc1be0da24494ff29c82bf
C'16_even = a4e7ba31b470bfff0d324405def8bc48
C'16_odd = 3baefc3253bbd339459fc3c1e0298ba0
C'17_even = e5c905fdf7ae090f947034124290f134
C'17_odd = a271b701e344ed95e93b8e364f2f984a
C'18_even = 88401d63a06cf61547c1444b8752afff
C'18_odd = 7ebb4af1e20ac6304670b6c5cc6e8ce6
C'19_even = a4d5a456bd4fca00da9d844bc83e18ae
C'19_odd = 7357ce453064d1ade8a6ce68145c2567
C'20_even = a3da8cf2cb0ee11633e906589a94999a
C'20_odd = 1f60b220c26f847bd1ceac7fa0d18518
C'21_even = 32595ba18ddd19d3509a1cc0aaa5b446
C'21_odd = 9f3d6367e4046bbaf6ca19ab0b56ee7e
C'22_even = 1fb179eaa9282174e9bdf7353b3651ee
C'22_odd = 1d57ac5a7550d3763a46c2fea37d7001
C'23_even = f735c1af98a4d84278edec209e6b6779
C'23_odd = 41836315ea3adba8fac33b4d32832c83
C'24_even = a7403b1f1c2747f35940f034b72d769a
C'24_odd = e73e4e6cd2214ffdb8fd8d39dc5759ef
C'25_even = 8d9b0c492b49ebda5ba2d74968f3700d
C'25_odd = 7d3baed07a8d5584f5a5e9f0e4f88e65
C'26_even = a0b8a2f436103b530ca8079e753eec5a
C'26_odd = 9168949256e8884f5bb05c55f8babc4c
C'27_even = e3bb3b99f387947b75daf4d6726b1c5d
C'27_odd = 64aeac28dc34b36d6c34a550b828db71
C'28_even = f861e2f2108d512ae3db643359dd75fc
C'28_odd = 1cacbcf143ce3fa267bbd13c02e843b0
C'29_even = 330a5bca8829a1757f34194db416535c
C'29_odd = 923b94c30e794d1e797475d7b6eeaf3f
C'30_even = eaa8d4f7be1a39215cf47e094c232751
C'30_odd = 26a32453ba323cd244a3174a6da6d5ad
C'31_even = b51d3ea6aff2c90883593d98916b3c56
C'31_odd = 4cf87ca17286604d46e23ecc086ec7f6
C'32_even = 2f9833b3b1bc765e2bd666a5efc4e62a
C'32_odd = 06f4b6e8bec1d43674ee8215bcef2163
C'33_even = fdc14e0df453c969a77d5ac406585826
C'33_odd = 7ec1141606e0fa167e90af3d28639d3f
C'34_even = d2c9f2e3009bd20c5faace30b7d40c30
C'34_odd = 742a5116f2e032980deb30d8e3cef89a
C'35_even = 4bc59e7bb5f17992ff51e66e048668d3
C'35_odd = 9b234d57e6966731cce6a6f3170a7505
```

